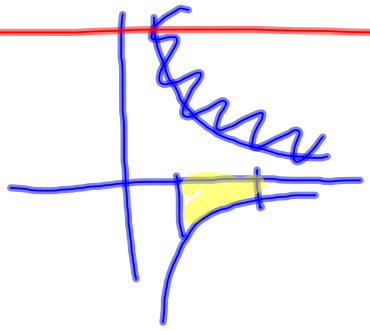
30. $\int_1^4 -x^{-2}\,dx = -\frac{x^{-1}}{-1}\Big|_1^4$

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$= \frac{1}{x}\Big|_1^4$

$= \frac{1}{4} - 1$

$= -\frac{3}{4}$



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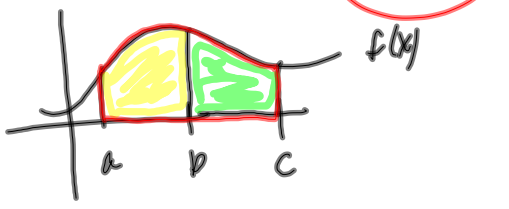
$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

$\int_a^a f(x)\,dx = 0$

$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$

$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



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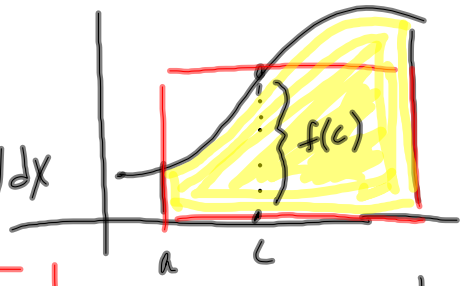
5.3 b MVT for definite integrals

Mean Value Theorem (average)

$(b-a)f(c) = \int_a^b f(x)\,dx$



Thm 3

$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

area of rectangle $= \int_a^b f(x)\,dx$

average value of $f(x)$ on $[a, b]$

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$f(x) = \sqrt{x}$ on $[0, 4]$

a) find the ~~average value~~

b) find c (the value that satisfies the MVT)

a) $\frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\int_0^4 x^{\frac{1}{2}}\,dx = \frac{1}{4}\cdot\frac{x^{3/2}}{3/2}\Big|_0^4$

$= \frac{1}{4}\cdot\frac{2}{3}\left(4^{3/2} - 0^{3/2}\right) = \frac{1}{6}\cdot 4^{3/2} = \frac{8}{6} = \frac{4}{3}$

b) $f(c) = \frac{4}{3}$

$\sqrt{c} = \frac{4}{3}$

$c = \frac{16}{9}$

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$f(x) = 4 - x^2 \quad [0, 3]$

a) find the average value of $f(x)$ on $[0,3]$

b) find the value that satisfies MVT

$$\frac{1}{3-0}\int_0^3 4 - x^2\,dx = \frac{1}{3}\left(4x - \frac{x^3}{3}\right)\Big|_0^3$$

$$= \frac{1}{3}\left(\left(12 - \frac{27}{3}\right) - (0 - 0)\right)$$

$$= \frac{1}{3}(12 - 9) = 1$$

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